

Physics 1A Discussion (Week 4)

Problem 1 (problem 3.44 on page 95 of the textbook)

CP CALC A dog in an open field is at rest under a tree at time $t = 0$ and then runs with acceleration

$$\vec{a}(t) = (0.400 \text{ m/s}^2) \hat{i} - (0.180 \text{ m/s}^3) t \hat{j}. \quad (1)$$

How far is the dog from the tree 8.00 s after it starts to run?

$$\begin{aligned} \vec{v}(t) &= \vec{v}_0 + \int_0^t dt \vec{a}(t) \\ &= \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \int_0^t dt \begin{pmatrix} (0.400 \text{ m/s}^2) \\ (-0.180 \text{ m/s}^3) t \end{pmatrix} \\ &= \begin{pmatrix} (0.400 \text{ m/s}^2) t \\ \frac{1}{2} (-0.180 \text{ m/s}^3) t^2 \end{pmatrix} \\ &= \begin{pmatrix} (0.400 \text{ m/s}^2) t \\ (-0.090 \text{ m/s}^3) t^2 \end{pmatrix} \end{aligned} \quad (2)$$

Then,

$$\begin{aligned} \vec{r}(t) &= \vec{r}_0 + \int_0^t dt \vec{v}(t) \\ &= \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \int_0^t dt \begin{pmatrix} (0.400 \text{ m/s}^2) t \\ (-0.090 \text{ m/s}^3) t^2 \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{2} (0.400 \text{ m/s}^2) t^2 \\ \frac{1}{3} (-0.090 \text{ m/s}^3) t^3 \end{pmatrix} \\ &= \begin{pmatrix} (0.200 \text{ m/s}^2) t^2 \\ (-0.030 \text{ m/s}^3) t^3 \end{pmatrix} \end{aligned} \quad (3)$$

At $t = 8$ s, the displacement vector becomes

$$\begin{aligned} \vec{r}(t = 8 \text{ s}) &= \begin{pmatrix} (0.200 \text{ m/s}^2) (8 \text{ s})^2 \\ (-0.030 \text{ m/s}^3) (8 \text{ s})^3 \end{pmatrix} \\ &= \begin{pmatrix} 1.28 \text{ m} \\ -15.36 \text{ m} \end{pmatrix}. \end{aligned} \quad (4)$$

Therefore, the displacement from the tree is given by $|\vec{r}(t = 8 \text{ s})| \sim 20.0 \text{ m}$.

Problem 2(problem 3.78 on page 98 of the textbook)

DATA You are a member of a geological team in Central Africa. Your team comes upon a wide river that is flowing east. You must determine the width of the river and the current speed (the speed of the water relative to the earth). You have a small boat with an outboard motor. By measuring the time it takes to cross a pond where the water isn't flowing, you have calibrated the throttle settings to the speed of the boat in still water. You set the throttle so that the speed of the boat relative to the river is a constant 6.00 m/s. Traveling due north across the river, you reach the opposite bank in 20.1 s. For the return trip, you change the throttle setting so that the speed of the boat relative to the river is 9.00 m/s. You travel due south from one bank to the other and cross the river in 11.2 s.

(a) How wide is the river, and what is the current speed?

Let v be the velocity of the river, and d be the width of the river. Then each trip satisfies that

$$[(6.00 \text{ m/s}) (20.1 \text{ s})]^2 = d^2 + [(20.1 \text{ s}) v]^2 \quad (5)$$

$$[(9.00 \text{ m/s}) (11.2 \text{ s})]^2 = d^2 + [(11.2 \text{ s}) v]^2. \quad (6)$$

Then, if you subtract the lower equation from the upper equation on both sides,

$$[(6.00 \text{ m/s}) (20.1 \text{ s})]^2 - [(9.00 \text{ m/s}) (11.2 \text{ s})]^2 = [(20.1 \text{ s}) v]^2 - [(11.2 \text{ s}) v]^2 \quad (7)$$

$$\begin{aligned} [(6.00 \text{ m/s}) (20.1 \text{ s}) + (9.00 \text{ m/s}) (11.2 \text{ s})] [(6.00 \text{ m/s}) (20.1 \text{ s}) - (9.00 \text{ m/s}) (11.2 \text{ s})] \\ = [(20.1 \text{ s}) v + (11.2 \text{ s}) v] [(20.1 \text{ s}) v - (11.2 \text{ s}) v] \end{aligned} \quad (8)$$

$$(221.4 \text{ m}) (19.8 \text{ m}) = (31.3 \text{ s}) (8.9 \text{ s}) v^2$$

$$v^2 \simeq 15.74 \text{ m}^2 / \text{s}^2$$

$$v \simeq 3.97 \text{ m/s} \quad (9)$$

$$(10)$$

Then if you plug this result into either Eq. (5) or Eq. (6),

$$d^2 = (100.8 \text{ m})^2 - (1973 \text{ m}^2)$$

$$\simeq 8187.64$$

$$d \simeq 90.5 \text{ m} \quad (11)$$

(b) With the throttle set so that the speed of the boat relative to the still water is 6.00 m/s, what is the shortest time in which you could cross the river, and where on the far bank would you land?

The traveling time from one side to the other is given by

$$t = \frac{d}{6.00 \text{ m/s}} \simeq 15.1 \text{ s}. \quad (12)$$

Then the boat reaches $vt \sim 3.97 \text{ m/s} \times 15.1 \text{ s} \simeq 59.9 \text{ m}$ east of the other side.

Problem 3 (Leaping the River II: problem 3.63 on page 97 of the textbook)

(Leaping the River II) A physics professor did daredevil stunts in his spare time. His last stunt was an attempt to jump across a river on a motorcycle (Fig. P3.63). The takeoff ramp was inclined at 53° , the river was 40.0 m wide, and the far bank was 15.0 m lower than the top of the ramp. The river itself was 100 m below the ramp. Ignore air resistance.

(a) What should his speed have been at the top of the ramp to have just made it to the edge of the far bank?

Let v_1 be the speed of the rider at the top of the ramp, and v_1^{minimum} be the minimum velocity to reach the other side of the cliff. We define v_1^{minimum} such that the rider can make it to the edge of the far bank. If you set the coordinate so that the top of the ramp is specified by (0, 15.0 m), the time evolutions of the horizontal and the vertical component of the rider's position are given by

$$x(t) = (v_1 \cos(53^\circ)) t, \quad (13)$$

$$y(t) = 15.0 \text{ m} + (v_1 \sin(53^\circ)) t - \frac{1}{2}gt^2. \quad (14)$$

At time $t = t_1 \equiv \frac{40.0 \text{ m}}{v_1 \cos(53^\circ)}$, it is necessary to require that

$$y(t_1) \geq 0. \quad (15)$$

This gives $v_1^2 \geq 320.41$. Then

$$v_1 \geq 17.9 \text{ m/s} = v_1^{\text{minimum}}. \quad (16)$$

(b) If his speed was only half the value found in part (a), where did he land?

In this problem, the initial speed of the bike is given by $v_2 = \frac{1}{2}v_1 \simeq 8.95 \text{ m/s}$. Then the evolution equation is modified by

$$x(t) = (v_2 \cos(53^\circ)) t, \quad (17)$$

$$y(t) = 15.0 \text{ m} + (v_2 \sin(53^\circ)) t - \frac{1}{2}gt^2. \quad (18)$$

Suppose at time $t = t_2$, the rider reaches a height as high as the river surface:

$$y(t_2) = 15.0 \text{ m} + (v_2 \sin(53^\circ)) t_2 - \frac{1}{2}gt_2^2 = -85 \text{ m}$$

$$\frac{1}{2}gt_2^2 - (v_2 \sin(53^\circ)) t_2 - 100 \text{ m} = 0 \text{ m}.$$

$$\left(4.9 \text{ m/s}^2\right) t_2^2 - (7.07 \text{ m/s}) t_2 - 100 \text{ m} = 0 \text{ m}. \quad (19)$$

$$\Rightarrow t_2 \simeq 5.30 \text{ s}. \quad (20)$$

At time $t = t_2$, the horizontal position of the rider is given by

$$\begin{aligned} x(t_2) &= (v_2 \cos(53^\circ)) t_2 \\ &\simeq 28.5 \text{ m} \leq 40.0 \text{ m}. \end{aligned} \quad (21)$$

Then, the bike lands in the river 28.5 m in front of the ramp.